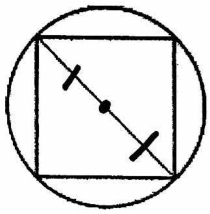
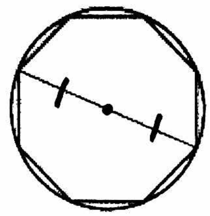
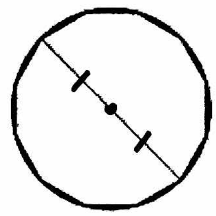
Name: Date: Period: 5

Circumference and Arc Length

Lesson Objective IWBAT find lengths of arcs in ⊙s; convert ∠ measures between degrees + radians.

Circumference: The circumference of a circle is the distance around the circle.

Consider a regular polygon inscribed in a circle. As the number of sides increases, the polygon approximates the circle and the ratio of the perimeter of the polygon to the diameter of the circle approaches π.

$\frac{\text{perim.}}{d} \approx \pi$

For all circles, the ratio of the circumference C to the diameter d is the same. This ratio is $\frac{C}{d} = \pi$.

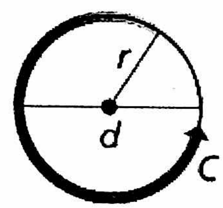
Solving for C yields the formula for the circumference of a circle, $C = \pi d$

Because $d = 2r$, you can write the formula as $C = 2\pi r$

Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$,

where d is the diameter of the circle and r is the radius of the circle.



Examples:

1. Find the circumference of a circle with a radius of 9 cm.

$C = 2\pi(9) = 18\pi$ cm (exact)

≈ 56.55 cm (approx.)

2. Find the radius of a circle with a circumference of 26 m.

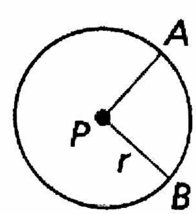
$26 = 2\pi r$

$r = \frac{13}{\pi}$ m ≈ 4.14 m

Arc Length: An arc length is a portion of the circumference of a circle. You can measure of the arc (in degrees) and find its length (in linear units).

Arc Length

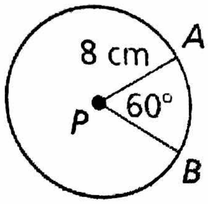
In a circle, the ratio of the length of a given arc to the circumference of a circle is equal to the ratio of the measure of the arc to 360°.



$$\frac{\text{arc measure}}{360^\circ} = \frac{\text{arc length}}{2\pi r}$$

Example:

3. Find the length of $\widehat{AB}$.

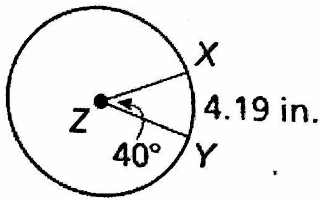


$$\frac{\cancel{60^\circ}\,1}{6\,\cancel{360^\circ}} = \frac{x}{2\pi(8)}$$

$$6x = 16\pi$$

$$\boxed{x = \frac{8\pi}{3}\text{ cm} \approx 8.38\text{ cm}}$$

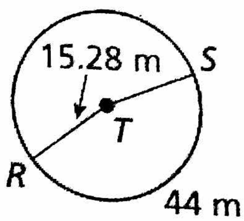
4. Find the circumference of $\odot Z$.



$$\frac{\cancel{40}\,1}{9\,\cancel{360}} = \frac{4.19}{C}$$

$$\boxed{C = 37.71\text{ in}}$$

5. Find $m\widehat{RS}$.



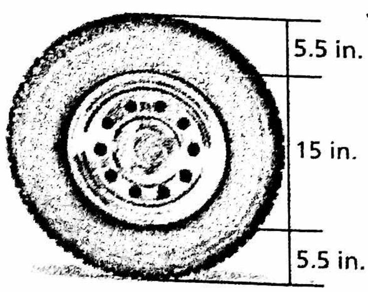
$$\frac{x}{360} = \frac{44}{2\pi(15.28)}$$

$$30.56\pi x = 15840$$

$$x = \frac{99000}{191\pi}$$

$$\boxed{x \approx 164.99^\circ}$$

6. The dimensions of a car tire are shown. To the nearest foot, how far does the tire travel when it makes 15 revolutions?



$d = 26$

1 rev $= \pi(26) = 26\pi$

15 revs $= 15(26\pi) = 390\pi$

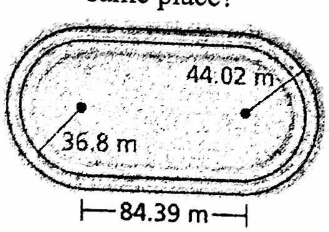
≈ 1225.22 in

$\boxed{1225\text{ in}}$

102.10 ft

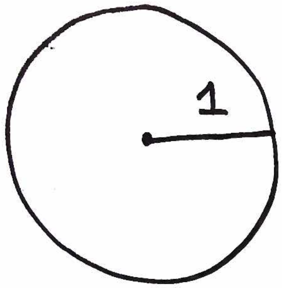
$\boxed{102\text{ ft}}$

7. The curves at the ends of a race track shown are 180° arcs of circles. The radius of the arc for a runner on the inside track is 36.8 meters. The radius of the arc for a runner on the outside track is 44.02 meters. How much further does the runner on the outside track run compared to the runner on the inside track if they start in the same place?



$$2(44.02)(\pi) - 2(36.8)(\pi) \quad \boxed{\approx 45.36\text{ m}}$$

Measuring Angles in Radians



circumf. of ⊙ w/ r=1 : $2\pi(1) = 2\pi$

degrees in a ⊙ : $360°$

$$45° : \frac{45}{360} = \frac{x}{2\pi} \to x = \frac{45}{\cancel{360}_{180}} \cdot \cancel{2}\pi = 45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$$

$$60° : \frac{60°}{360} = \frac{x}{2\pi} \to x = \frac{60}{360} \cdot 2\pi = 60 \cdot \frac{\pi}{180} = \frac{\pi}{3}$$

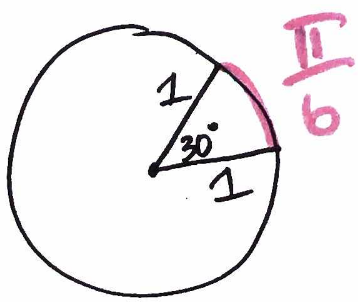
Converting between Degrees and Radians

Degrees to Radians	*Radians to Degrees*
Multiply degree measure by… $\frac{2\pi}{360} \to \frac{\pi}{180}$	Multiply radian measure by… $\frac{180}{\pi}$

Example:

8. Convert ~~45°~~ 30° to radians.

$$30 \times \frac{\pi}{180} = \frac{\pi}{6}$$



9. Convert $\frac{3\pi}{2}$ radians to degrees

$$\frac{3\cancel{\pi}}{\cancel{2}} \cdot \frac{\overset{90}{\cancel{180}}}{\cancel{\pi}} = 270°$$